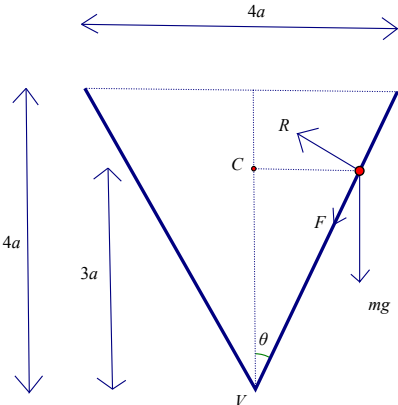
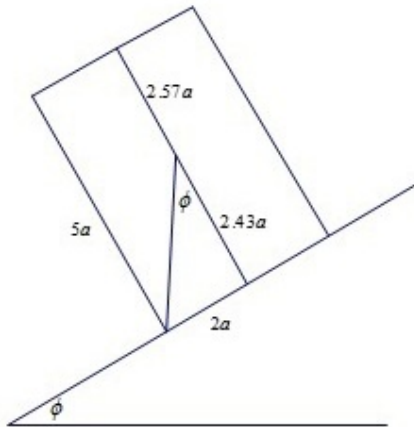


Paper 4F: Further Mechanics 2 Mark Scheme

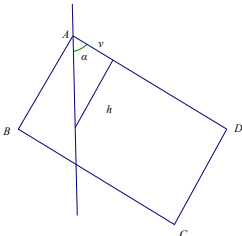
Question	Scheme	Marks	AOs
1(a)	Total mass = $\int_0^{15} 10\left(1 - \frac{x}{25}\right) dx$	M1	2.1
	$= \left[10x - \frac{x^2}{5} \right]_0^{15}$	A1	1.1b
	$= 150 - \frac{225}{5} = 105 \text{ (kg) } *$	A1*	1.1b
		(3)	
(b)	Taking moments about the base: $\int_0^{15} 10x\left(1 - \frac{x}{25}\right) dx$	M1	3.4
	$= \left[5x^2 - \frac{2}{15}x^3 \right]_0^{15} (= 675)$	A1	1.1b
	$\Rightarrow 105d = 675$	M1	3.4
	$d = 6.43 \text{ (m) } \quad 6\frac{3}{7} \text{ (m)}$	A1	1.1b
		(4)	
(7 marks)			
Notes:			
(a) M1: Use integration (usual rules) A1: Correct integration A1*: Use limits and show sufficient working to justify given answer			
(b) M1: Use the model to find the moment about the base (usual rules for integration) A1: Correct integration M1: Use the model to complete the moments equation Require 105 and their 675 used correctly A1: 6.43 or better			

Question	Scheme	Marks	AOs
2			
	Complete overall strategy	M1	3.1b
	Resolve vertically	M1	3.3
	$mg + F \cos \theta = R \sin \theta$	A1	1.1b
	Horizontal equation of motion	M1	3.3
	$mr\omega^2 = R \cos \theta + F \sin \theta$	A1	1.1b
	Use of limiting friction since maximum ω	M1	3.3
	Substitute for trig ratios: $\frac{3a\omega^2}{2g} = \frac{9}{2}$	M1	1.1b
	Maximum $\omega = \sqrt{\frac{3g}{a}}$	A1	1.1b
(8 marks)			
Notes:			
<p>M1: Overall strategy to form equation in ω only e.g. consider vertical and horizontal motion and limiting friction</p> <p>M1: Needs all 3 terms. Condone sign errors and sin/cos confusion</p> <p>A1: Correct unsimplified equation</p> <p>M1: Needs all 3 terms. Condone sign errors and sin/cos confusion</p> <p>A1: Correct unsimplified equation</p> <p>M1: Seen or implied</p> <p>M1: Substitute to achieve equation in a, ω and g only</p> <p>A1: Or equivalent exact form</p>			

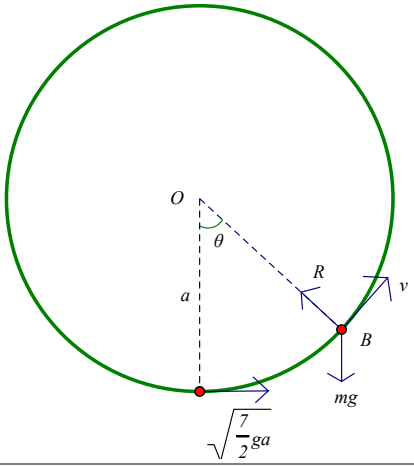
Question	Scheme	Marks	AOs												
3(a)	<table><tr><td></td><td>mass</td><td>c of m from O</td></tr><tr><td>cylinder</td><td>$4\pi a^2 h$</td><td>$\frac{h}{2}$</td></tr><tr><td>hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr><tr><td>V</td><td>$4\pi a^2 h - \frac{2}{3}\pi a^3$</td><td>$d$</td></tr></table>		mass	c of m from O	cylinder	$4\pi a^2 h$	$\frac{h}{2}$	hemisphere	$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$	V	$4\pi a^2 h - \frac{2}{3}\pi a^3$	d		
		mass	c of m from O												
	cylinder	$4\pi a^2 h$	$\frac{h}{2}$												
	hemisphere	$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$												
	V	$4\pi a^2 h - \frac{2}{3}\pi a^3$	d												
	Mass ratios	B1	1.2												
	Correct distances	B1	1.2												
Moments about a diameter through O	M1	2.1													
$4\pi a^2 h \times \frac{h}{2} - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = 2\pi a^2 \left(2h - \frac{1}{3}a\right) \times d$	A1	1.1b													
$d = \frac{h^2 - \frac{a^2}{8}}{2h - \frac{a}{3}} = \frac{3(8h^2 - a^2)}{8(6h - a)} \quad *$	A1*	2.2a													
	(5)														
(b)															
	$h = 5a \Rightarrow d = 2.573...a$	B1	1.1b												
	About to topple so c of m above tipping point	M1	2.2a												
	$\Rightarrow \tan \phi = \frac{2a}{5a - 2.573a}$	A1ft	1.1b												
	$\phi = 39.5^\circ$ or 0.689 rads	A1	1.1b												
		(4)													
(9 marks)															

Notes:**(a)****B1:** Correct mass ratios**B1:** Correct distances**M1:** All three terms & dimensionally correct. Could use a parallel axis but final answer must be for the distance from O **A1:** Correct unsimplified equation**A1*:** Deduce the given answer. Their working must make it clear how they reached their answer**(b)****B1:** Distance of com from base**M1:** Condone tan the wrong way up**A1ft:** Correct unsimplified expression for trig ratio for ϕ following their d **A1:** 39.5° or 0.689 rads

Question	Scheme	Marks	AOs
4(a)	Equation of motion: $1800 - 2v^2 = 500a$ (when seen)	B1	2.1
	Select form for a : $= 500 \frac{dv}{dt}$	M1	2.5
	$\int \frac{2}{500} dt = \int \frac{1}{900 - v^2} dv = \frac{1}{60} \int \frac{1}{30 + v} + \frac{1}{30 - v} dv$	M1	2.1
	$\frac{t}{250} = \frac{1}{60} \ln(30 + v) - \frac{1}{60} \ln(30 - v) (+C)$	A1	1.1b
	$T = \frac{25}{6} \ln \left(\frac{30 + 10}{30 - 10} \right) = \frac{25}{6} \ln 2$ *	M1 A1*	2.1 2.2a
		(6)	
(b)	Equation of motion: $500v \frac{dv}{dx} = 1800 - 2v^2$	M1	2.5
	$\int \frac{500v}{1800 - 2v^2} dv = \int 1 dx$	M1	2.1
	$-125 \ln(1800 - 2v^2) = x (+C)$	A1	1.1b
	Use boundary conditions: $x = -125 \ln 1600 + 125 \ln 1800$	M1	2.1
	$x = 125 \ln \frac{9}{8} \text{ (m)}$ *	A1*	2.2a
		(5)	
(11 marks)			
Notes:			
(a) B1: All three terms & dimensionally correct M1: Use of correct form for acceleration to give equation in v, t only M1: Separate variables and integrate A1: Condone missing C M1: Use boundary conditions correctly A1*: Show sufficient working to justify given answer and a 'statement' that the required result has been achieved			
(b) M1: Correct form of acceleration in the equation of motion to give equation in v, x only M1: Separate variables and integrate A1: Condone missing C M1: Extract and use boundary conditions A1*: Show sufficient working to justify given answer and a 'statement' that the required result has been achieved			

Question	Scheme	Marks	AOs												
5(a)	<table><tr><td></td><td>Mass</td><td>From AD</td></tr><tr><td>Rectangle</td><td>$8a^2$</td><td>a</td></tr><tr><td>Semicircle</td><td>$\frac{1}{2}\pi a^2$</td><td>$\frac{4a}{3\pi}$</td></tr><tr><td>Sign</td><td>$a^2\left(8-\frac{\pi}{2}\right)$</td><td>$h$</td></tr></table>		Mass	From AD	Rectangle	$8a^2$	a	Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$	Sign	$a^2\left(8-\frac{\pi}{2}\right)$	h		
		Mass	From AD												
	Rectangle	$8a^2$	a												
	Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$												
	Sign	$a^2\left(8-\frac{\pi}{2}\right)$	h												
	Mass ratios	B1	1.2												
Moments about AD	M1	2.1													
$a^2\left(8-\frac{\pi}{2}\right)h=8a^2\times a-\frac{1}{2}\pi a^2\times\frac{4a}{3\pi}\left(=8a^3-\frac{2}{3}a^3=\frac{22}{3}a^3\right)$	A1	1.1b													
$\Rightarrow h=\frac{22}{3}a\div\left(8-\frac{\pi}{2}\right)=\frac{44a}{3(16-\pi)}\quad *$	A1*	2.2a													
	(4)														
(b)	Moments about A $2aT=\frac{44a}{3(16-\pi)}W$	M1	3.1b												
	$T=\frac{hW}{2a}=\frac{22W}{3(16-\pi)}$	A1	1.1b												
		(2)													
(c)															
	Take moments about AB to find distance of com from AB	M1	3.1b												
	$8a^2\times 2a-\frac{1}{2}\pi a^2\times d=\left(8-\frac{1}{2}\pi\right)a^2\times v$	A1	1.1b												
	$v=\frac{32a-\pi d}{16-\pi}$	A1	1.1b												
	Correct trig for the given angle	M1	3.1b												
	$\tan\alpha=\frac{11}{18}=\frac{h}{v}=\frac{44a}{3(32a-\pi d)}$	A1ft	1.1b												
	$(24a=32a-\pi d,\quad 8a=\pi d)\quad d=\frac{8a}{\pi}$	A1	1.1b												
		(6)													
(12 marks)															

Notes:	
(a)	<p>B1: Correct mass ratios</p> <p>M1: Need all three terms, must be dimensionally correct</p> <p>A1: Correct unsimplified equation</p> <p>A1*: Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved</p>
(b)	<p>M1: Could also take moments about B or about the c.o.m. and use</p> <p>A1: cso</p>
(c)	<p>M1: All terms and dimensionally correct</p> <p>A1: Correct unsimplified equation</p> <p>A1: Or equivalent</p> <p>M1: Condone tan the wrong way up</p> <p>A1: Equation in a and d; follow through on their v</p> <p>A1: cao</p>

Question	Scheme	Marks	AOs
6(a)			
	Conservation of energy	M1	2.1
	$\frac{1}{2}mv^2 + mga(1 - \cos \theta) = \frac{1}{2}m\left(\frac{7}{2}ga\right)$	A1	1.1b
	$v^2 = ga\left(\frac{3}{2} + 2\cos \theta\right)^*$	A1*	2.2a
		(3)	
(b)	Resolve parallel to OB and use $\frac{mv^2}{a}$	M1	3.1b
	$R - mg \cos \theta = \frac{mv^2}{a}$	A1	1.1b
	Use $R = 0$ $g \cos \theta = -\frac{v^2}{a}$	M1	3.1b
	Solve for $\theta \Rightarrow g \cos \theta = -g\left(\frac{3}{2} + 2\cos \theta\right)$	M1	1.1b
	$\theta = 120^\circ$	A1	1.1b
		(5)	
(c)	Any appropriate comment e.g. the hoop is unlikely to be smooth	B1	3.5b
		(1)	

Question	Scheme	Marks	AOs
(d)	At rest $\Rightarrow v = 0$	M1	3.1b
	$\Rightarrow \cos \theta = -\frac{3}{4}$	A1	1.1b
	Acceleration is tangential	M1	3.1b
	Magnitude $ g \cos(\theta - 90) = 6.48 \text{ m s}^{-2}$ or $\frac{\sqrt{7}}{4}g$	A1	1.1b
	At $\left(\cos^{-1}\left(-\frac{3}{4}\right) - 90 = \right) 48.6^\circ$ to the downward vertical	A1	1.1b
		(5)	
(14 marks)			
Notes:			
(a) M1: All terms required. Must be dimensionally correct A1: Correct unsimplified equation A1*: Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved			
(b) M1: Resolve parallel to OB A1: Correct equation M1: Use $R = 0$ seen or implied M1: Solve for θ A1: Accept $\frac{2\pi}{3}$			
(c) B1: Any appropriate comment e.g. - hoop may not be smooth; - air resistance could affect the motion			
(d) M1: $v = 0$ seen or implied A1: Correct equation in θ M1: Correct direction for acceleration A1: Accept 6.48, 6.5 or exact in g A1: Accept 0.848 (radians)			

Question	Scheme	Marks	AOs
7(a)			
	$T_A = \frac{20e}{2}, T_B = \frac{50(2-e)}{2} e$	M1	3.1a
	In equilibrium $T_A = T_B, 10e = 25(2-e)$	M1	3.1a
	$(35e = 50), e = \frac{10}{7}$	A1	1.1b
	Equation of motion for P when distance x from equilibrium position towards B :	M1	3.1a
	$3.5\ddot{x} = T_B - T_A = \frac{50(2-e-x)}{2} - \frac{20(e+x)}{2}$	A1 A1	1.1b 1.1b
	$= \frac{50\left(\frac{4}{7}-x\right)}{2} - \frac{20\left(\frac{10}{7}+x\right)}{2}$		
	$\Rightarrow 3.5\ddot{x} = -35x, \ddot{x} = -10x$ and hence SHM about the equilibrium position	A1	3.2a
		(7)	
(b)	Amplitude $= 2 - \frac{10}{7} = \frac{4}{7}$	B1 ft	2.2a
	Use of max speed $= a\omega$	M1	1.1b
	$= \frac{4}{7}\sqrt{10} = 1.81 \text{ (m s}^{-1}\text{)}$	A1 ft	1.1b
		(3)	

Question	Scheme	Marks	AOs
7(c)	Nearer to A than to B : $x < -\frac{3}{7}$	B1	3.1a
	Solve for $\sqrt{10}t$: $\cos \sqrt{10}t = -\frac{3}{4}$, $\sqrt{10}t = 2.418\dots\dots\dots$	M1	3.1a
	Length of time: $\frac{2}{\sqrt{10}}(\pi - 2.418\dots)$	M1	1.1b
	0.457 (seconds)	A1	1.1b
	Alternative: $\frac{3.864 - 2.419}{\sqrt{10}} = 0.457$		
	Alternative: $x = \frac{4}{7} \sin \sqrt{10}t = \frac{3}{7} \Rightarrow \sqrt{10}t = 0.8481 \text{ or } \sqrt{10}t = 2.29353$ $t_1 = 0.2682, t_2 = 0.72527$ $\Rightarrow \text{time} = 0.457 \text{ (seconds)}$		
		(4)	
(14 marks)			
Notes:			
<p>(a)</p> <p>M1: Use of $T = \frac{\lambda x}{a}$</p> <p>M1: Dependent on the preceding M1. Equate their tensions</p> <p>A1: cao</p> <p>M1: Condone sign error</p> <p>A1: Correct unsimplified equation in e and x A1A1 Equation with one error A1A0</p> <p>A1: Full working to justify conclusion that it is SHM about the equilibrium position</p>			
<p>(b)</p> <p>B1ft: Seen or implied. Follow their e</p> <p>M1: Correct method for max. speed</p> <p>A1ft: 1.81 or better. Follow their a, ω</p>			
<p>(c)</p> <p>B1: Seen or implied</p> <p>M1: Use of $x = a \cos \omega t$</p> <p>M1: Correct strategy for the required interval</p> <p>A1: 0.457 or better</p>			