Paper 1: Core Pure Mathematics 1 Mark Scheme

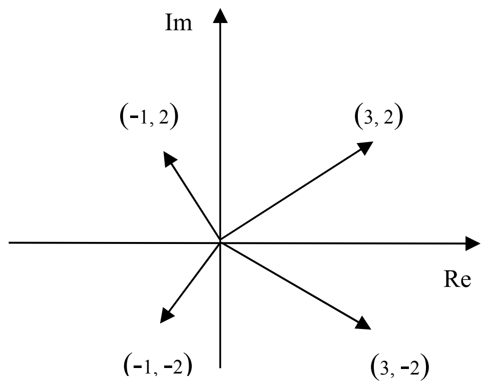
Question	Scheme	Marks	AOs
1	$\frac{1}{(r+1)(r+3)} \equiv \frac{A}{(r+1)} + \frac{B}{(r+3)} \Rightarrow A = ..., B = ...$	M1	3.1a
	$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} =$ $\frac{1}{2\times2} - \frac{1}{2\times4} + \frac{1}{2\times3} - \frac{1}{2\times5} + ... + \frac{1}{2n} - \frac{1}{2(n+2)} + \frac{1}{2(n+1)} - \frac{1}{2(n+3)}$	M1	2.1
	$= \frac{1}{4} + \frac{1}{6} - \frac{1}{2(n+2)} - \frac{1}{2(n+3)}$	A1	2.2a
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{12(n+2)(n+3)}$	M1	1.1b
	$= \frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
		(5)	
	Alternative by induction: $n = 1 \Rightarrow \frac{1}{8} = \frac{a+b}{12\times3\times4},\ n = 2 \Rightarrow \frac{1}{8} + \frac{1}{15} = \frac{2(2a+b)}{12\times4\times5}$ $a + b = 18,\ 2a + b = 23 \Rightarrow a = ..., b = ...$	M1	3.1a
	Assume true for $n = k$ so $\sum_{r=1}^{k} \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)}$		
	$\sum_{r=1}^{k+1} \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)}$	M1	2.1
	$\frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)} = \frac{k(5k+13)(k+4) + 12(k+3)}{12(k+2)(k+3)(k+4)}$	A1	2.2a
	$= \frac{5k^3 + 33k^2 + 52k + 12k + 36}{12(k+2)(k+3)(k+4)} = \frac{(k+1)(k+2)(5k+18)}{12(k+2)(k+3)(k+4)}$	M1	1.1b
	$= \frac{(\underline{k+1})(5(\underline{k+1})+13)}{12(\underline{k+1}+2)(\underline{k+1}+3)}$ So true for $n = k + 1$ So $\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} = \frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
		(5)	
			(5 marks)

Question 1 notes:
Main Scheme **M1:** Valid attempt at partial fractions **M1:** Starts the process of differences to identify the relevant fractions at the start and end **A1:** Correct fractions that do not cancel **M1:** Attempt common denominator **A1:** Correct answer
Alternative by Induction: **M1:** Uses $n = 1$ and $n = 2$ to identify values for a and b **M1:** Starts the induction process by adding the $(k + 1)^{\text{th}}$ term to the sum of k terms **A1:** Correct single fraction **M1:** Attempt to factorise the numerator **A1:** Correct answer and conclusion

Question	Scheme	Marks	AOs
2	When $n = 1$, $2^{3n+1} + 3\left(5^{2n+1}\right) = 16 + 375 = 391$ $391 = 17 \times 23$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $2^{3k+1} + 3\left(5^{2k+1}\right)$ is divisible by 17	M1	2.4
	$\mathrm{f}\left(k+1\right) - \mathrm{f}\left(k\right) = 2^{3k+4} + 3\left(5^{2k+3}\right) - 2^{3k+1} - 3\left(5^{2k+1}\right)$	M1	2.1
	$= 7 \times 2^{3k+1} + 7 \times 3\left(5^{2k+1}\right) + 17 \times 3\left(5^{2k+1}\right)$		
	$= 7\mathrm{f}\left(k\right) + 17 \times 3\left(5^{2k+1}\right)$	A1	1.1b
	$\mathrm{f}\left(k+1\right) = 8\mathrm{f}\left(k\right) + 17 \times 3\left(5^{2k+1}\right)$	A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	A1	2.4
		(6)	
			(6 marks)

Notes:

B1: Shows the statement is true for $n = 1$
M1: Assumes the statement is true for $n = k$
M1: Attempts $\mathrm{f}(k+1) - \mathrm{f}(k)$
A1: Correct expression in terms of $\mathrm{f}(k)$
A1: Correct expression in terms of $\mathrm{f}(k)$
A1: Obtains a correct expression for $\mathrm{f}(k + 1)$
A1: Correct complete conclusion

Question	Scheme	Marks	AOs
3	$z = 3 - 2\mathrm{i}$ is also a root	B1	1.2
	$\left(z-(3+2\mathrm{i})\right)\left(z-(3-2\mathrm{i})\right) = \ldots$ **or** Sum of roots = 6, Product of roots = 13 $\Rightarrow \ldots$	M1	3.1a
	$= z^2 - 6z + 13$	A1	1.1b
	$\left(z^4 + az^3 + 6z^2 + bz + 65\right) = \left(z^2 - 6z + 13\right)\left(z^2 + cz + 5\right) \Rightarrow c = \ldots$	M1	3.1a
	$z^2 + 2z + 5 = 0$	A1	1.1b
	$z^2 + 2z + 5 = 0 \Rightarrow z = \ldots$	M1	1.1a
	$z = -1 \pm 2\mathrm{i}$	A1	1.1b
		B1 $3 \pm 2\mathrm{i}$ Plotted correctly	1.1b
		B1ft $-1 \pm 2\mathrm{i}$ Plotted correctly	1.1b
			(9 marks)

Notes:

B1: Identifies the complex conjugate as another root
M1: Uses the conjugate pair and a correct method to find a quadratic factor
A1: Correct quadratic
M1: Uses the given quartic and their quadratic to identify the value of c
A1: Correct 3TQ
M1: Solves their second quadratic
A1: Correct second conjugate pair
B1: First conjugate pair plotted correctly and labelled
B1ft: Second conjugate pair plotted correctly and labelled (Follow through their second conjugate pair)

Question	Scheme	Marks	AOs
4	$4+\cos 2\theta=\frac{9}{2}\Rightarrow\theta=...$	M1	3.1a
	$\theta=\frac{\pi}{6}$	A1	1.1b
	$\frac{1}{2}\int(4+\cos 2\theta)^2\mathrm{d}\theta=\frac{1}{2}\int(16+8\cos 2\theta+\cos^2 2\theta)\mathrm{d}\theta$	M1	3.1a
	$\cos^2 2\theta=\frac{1}{2}+\frac{1}{2}\cos 4\theta\Rightarrow A=\frac{1}{2}\int\left(16+8\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)\mathrm{d}\theta$	M1	3.1a
	$=\frac{1}{2}\left[16\theta+4\sin 2\theta+\frac{\sin 4\theta}{8}+\frac{\theta}{2}\right]$	A1	1.1b
	Using limits 0 and their $\frac{\pi}{6}$: $\frac{1}{2}\left[\frac{33\pi}{12}+2\sqrt{3}+\frac{\sqrt{3}}{16}-(0)\right]$	M1	1.1b
	Area of triangle = $\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{81}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}$	M1	3.1a
	Area of $R=\frac{33\pi}{24}+\frac{33\sqrt{3}}{32}-\frac{81\sqrt{3}}{32}$	M1	1.1b
	$=\frac{11}{8}\pi-\frac{3\sqrt{3}}{2}\left(p=\frac{11}{8},\ q=-\frac{3}{2}\right)$	A1	1.1b
			(9 marks)

Notes:

M1: Realises the angle for A is required and attempts to find it
A1: Correct angle
M1: Uses a correct area formula and squares r to achieve a 3TQ integrand in $\cos 2\theta$
M1: Use of the correct double angle identity on the integrand to achieve a suitable form for integration
A1: Correct integration
M1: Correct use of limits
M1: Identifies the need to subtract the area of a triangle and so finds the area of the triangle
M1: Complete method for the area of R
A1: Correct final answer

Question	Scheme	Marks	AOs
5(a)	Pond contains 1000 + 5t litres after t days	M1	3.3
	If x is the amount of pollutant in the pond after t days Rate of pollutant out = $20\times\dfrac{x}{1000+5t}$ g per day	M1	3.3
	Rate of pollutant in = 25 × 2 g = 50g per day	B1	2.2a
	$\dfrac{dx}{dt}=50-\dfrac{4x}{200+t}$ *	A1*	1.1b
		(4)	
(b)	$I=e^{\int\frac{4}{200+t}dt}=(200+t)^4 \Rightarrow x(200+t)^4=\int 50(200+t)^4\,dt$	M1	3.1b
	$x(200+t)^4=10(200+t)^5+c$	A1	1.1b
	$x=0,\ t=0\Rightarrow\ c=-3.2\times10^{12}$	M1	3.4
	$t=8\Rightarrow\ x=\ 10(200+8)-\dfrac{3.2\times10^{12}}{(200+8)^4}$	M1	1.1b
	= 370g	A1	2.2b
		(5)	
(c)	e.g. • The model should take into account the fact that the pollutant does not dissolve throughout the pond upon entry • The rate of leaking could be made to vary with the volume of water in the pond	B1	3.5c
		(1)	
		(10 marks)	

Notes:

(a)
M1: Forms an expression of the form 1000 + kt for the volume of water in the pond at time t
M1: Expresses the amount of pollutant out in terms of x and t
B1: Correct interpretation for pollutant entering the pond
A1*: Puts all the components together to form the correct differential equation

(b)
M1: Uses the model to find the integrating factor and attempts solution of their differential equation
A1: Correct solution
M1: Interprets the initial conditions to find the constant of integration
M1: Uses their solution to the problem to find the amount of pollutant after 8 days
A1: Correct number of grams

(c)
B1: Suggests a suitable refinement to the model

Question	Scheme	Marks	AOs
6(a)	$\mathrm{f}(x)=\frac{x+2}{x^2+9}=\frac{x}{x^2+9}+\frac{2}{x^2+9}$	B1	3.1a
	$\int\frac{x}{x^2+9}\,\mathrm{d}x=k\ln(x^2+9)(+c)$	M1	1.1b
	$\int\frac{2}{x^2+9}\,\mathrm{d}x=k\arctan\left(\frac{x}{3}\right)(+c)$	M1	1.1b
	$\int\frac{x+2}{x^2+9}\,\mathrm{d}x=\frac{1}{2}\ln(x^2+9)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)+c$	A1	1.1b
		(4)	
(b)	$\int_0^3\mathrm{f}(x)\,\mathrm{d}x=\left[\frac{1}{2}\ln(x^2+9)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)\right]_0^3$ $=\frac{1}{2}\ln 18+\frac{2}{3}\arctan\left(\frac{3}{3}\right)-\left(\frac{1}{2}\ln 9+\frac{2}{3}\arctan(0)\right)$ $=\frac{1}{2}\ln\frac{18}{9}+\frac{2}{3}\arctan\left(\frac{3}{3}\right)$	M1	1.1b
	Mean value $=\frac{1}{3-0}\left(\frac{1}{2}\ln 2+\frac{\pi}{6}\right)$	M1	2.1
	$\frac{1}{6}\ln 2+\frac{1}{18}\pi$ *	A1*	2.2a
		(3)	
(c)	$\frac{1}{6}\ln 2+\frac{1}{18}\pi+\ln k$	M1	2.2a
	$\frac{1}{6}\ln 2k^6+\frac{1}{18}\pi$	A1	1.1b
		(2)	
			(9 marks)

Notes:

(a)
B1: Splits the fraction into two correct separate expressions
M1: Recognises the required form for the first integration
M1: Recognises the required form for the second integration
A1: Both expressions integrated correctly and added together with constant of integration included

(b)
M1: Uses limits correctly and combines logarithmic terms
M1: Correctly applies the method for the mean value for their integration
A1*: Correct work leading to the given answer

(c)
M1: Realises that the effect of the transformation is to increase the mean value by ln k
A1: Combines ln's correctly to obtain the correct expression

Question	Scheme	Marks	AOs
7(a)	$x = \cos\theta + \sin\theta\cos\theta = -y\cos\theta$	M1	2.1
	$\sin\theta = -y-1$	M1	2.1
	$\left(\frac{x}{-y}\right)^2 = 1-(-y-1)^2$	M1	2.1
	$x^2 = -\left(y^4+2y^3\right)$*	A1*	1.1b
		(4)	
(b)	$V = \pi\int x^2\,\mathrm{d}y = \pi\int -\left(y^4+2y^3\right)\,\mathrm{d}y$	M1	3.4
	$= \pi\left[-\left(\frac{y^5}{5}+\frac{y^4}{2}\right)\right]$	A1	1.1b
	$= -\pi\left[\left(\frac{(0)^5}{5}+\frac{(0)^4}{2}\right)-\left(\frac{(-2)^5}{5}+\frac{(-2)^4}{2}\right)\right]$	M1	3.4
	$= 1.6\pi\,\text{cm}^3$ **or** awrt 5.03 cm^3	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)
M1: Obtains x in terms of y and $\cos\theta$
M1: Obtains an equation connecting y and $\sin\theta$
M1: Uses Pythagoras to obtain an equation in x and y only
A1*: Obtains printed answer

(b)
M1: Uses the correct volume of revolution formula with the given expression
A1: Correct integration
M1: Correct use of correct limits
A1: Correct volume

Question	Scheme	Marks	AOs
8	$2+4\lambda-2(4-2\lambda)-6+\lambda=6 \Rightarrow \lambda=...$	M1	1.1b
	$\lambda=2 \Rightarrow$ Required point is $(2+2(4),\ 4+2(-2),\ -6+2(1))$ $(10,\ 0,\ -4)$	A1	1.1b
	$2+t-2(4-2t)-6+t=6 \Rightarrow t=...$	M1	3.1a
	$t=3$ so reflection of $(2,4,-6)$ is $(2+6(1),4+6(-2),-6+6(1))$	M1	3.1a
	$(8,\ -8,\ 0)$	A1	1.1b
	$\begin{pmatrix}10\\0\\-4\end{pmatrix}-\begin{pmatrix}8\\-8\\0\end{pmatrix}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$	M1	3.1a
	$\mathbf{r}=\begin{pmatrix}10\\0\\-4\end{pmatrix}+k\begin{pmatrix}1\\4\\-2\end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r}-\begin{pmatrix}10\\0\\-4\end{pmatrix}\right)\times\begin{pmatrix}1\\4\\-2\end{pmatrix}=\mathbf{0}$	A1	2.5
		(7)	
			(7 marks)

Notes:

M1: Substitutes the parametric equation of the line into the equation of the plane and solves for λ

A1: Obtains the correct coordinates of the intersection of the line and the plane

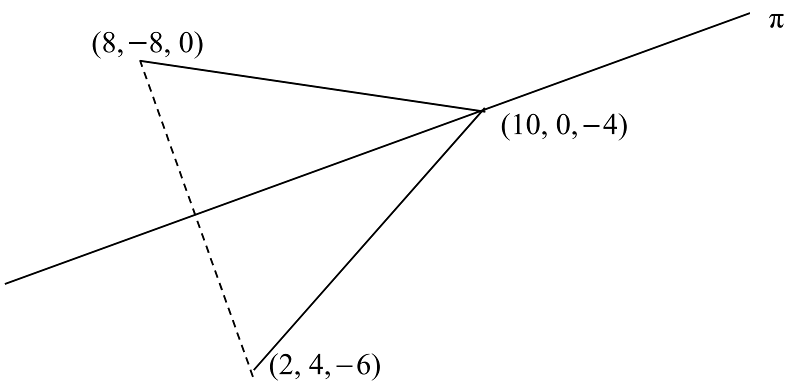
M1: Substitutes the parametric form of the line perpendicular to the plane passing through $(2,\ 4,\ -6)$ into the equation of the plane to find t

M1: Find the reflection of $(2,\ 4,\ -6)$ in the plane

A1: Correct coordinates

M1: Determines the direction of l by subtracting the appropriate vectors

A1: Correct vector equation using the correct notation



Question	Scheme	Marks	AOs
9(a)(i)	Weight = mass×g $\Rightarrow m = \dfrac{30\,000}{g} = 3000$ But mass is in thousands of kg, so $m = 3$	M1	3.3
(ii)	$\dfrac{dx}{dt} = 40\cos t + 20\sin t,\ \dfrac{d^2x}{dt^2} = -40\sin t + 20\cos t$	M1	1.1b
	$3(-40\sin t + 20\cos t) + 4(40\cos t + 20\sin t)$ $+40\sin t - 20\cos t = ...$	M1	1.1b
	$= 200\cos t$ so PI is $x = 40\sin t - 20\cos$	A1*	2.1
	or		
	Let $x = a\cos t + b\sin t$ $\dfrac{dx}{dt} = -a\sin t + b\cos t,\ \dfrac{d^2x}{dt^2} = -a\cos t - b\sin t$	M1	1.1b
	$4b - 2a = 200,\ -2b - 4a = 0 \Rightarrow a = ..., b = ...$	M1	2.1
	$x = 40\sin t - 20\cos t$	A1*	1.1b
(iii)	$3\lambda^2 + 4\lambda + 1 = 0 \Rightarrow \lambda = -1, -\dfrac{1}{3}$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t}$	A1	1.1b
	$x = PI + CF$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	A1	1.1b
		(8)	
(b)	$t = 0, x = 0 \Rightarrow A + B = 20$	M1	3.4
	$x = 0, \dfrac{dx}{dt} = -Ae^{-t} - \dfrac{1}{3}Be^{-\frac{1}{3}t} + 40\cos t + 20\sin t = 0$ $\Rightarrow A + \dfrac{1}{3}B = 40$	M1	3.4
	$x = 50e^{-t} - 30e^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	A1	1.1b
	$t = 9 \Rightarrow x = 33\text{m}$	A1	3.4
		(4)	
			(12 marks)

Question 9 notes:
(a)(i) **M1:** Correct explanation that in the model, $m = 3$
(ii) **M1:** Differentiates the given PI twice **M1:** Substitutes into the given differential equation **A1*:** Reaches 200cos$t$ and makes a conclusion **or** **M1:** Uses the correct form for the PI and differentiates twice **M1:** Substitutes into the given differential equation and attempts to solve **A1*:** Correct PI
(iii) **M1:** Uses the model to form and solve the auxiliary equation **A1:** Correct complementary function **M1:** Uses the correct notation for the general solution by combining PI and CF **A1:** Correct General Solution for the model
(b) **M1:** Uses the initial conditions of the model, $t = 0$ at $x = 0$, to form an equation in A and B **M1:** Uses $\frac{dx}{dt} = 0$ at $x = 0$ in the model to form an equation in A and B **A1:** Correct PS **A1:** Obtains 33m using the assumptions made in the model